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Universal Delineation of Particle Separation Systems and Separation Results of Stratified Material

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Abstract: Elements needed for a meaningful delineation of particle separation systems and prediction of separation results have been identified and characterized. The most important elements of particle separation systems are balance of forces or equivalent quantities such as energy, momentum, probability, etc., as well as particle trajectory, stratification of particles, and splitting the stratified material into products.

Keywords: Classification, particle trajectory, recovery, separation, separation forces, splitting, stratification, upgrading, yield

INTRODUCTION

Particle separation systems are interactive. They consist of a material, usually called the feed, containing components of different features, and separation device which provides external force fields, space, and time for separation. Separation system can be influenced by various factors causing changes of separation results (Fig. 1).

The components of the feed, due to their features, interact with the external force fields of the separator leading to appearance of ordering forces which create movement of the particles in the separator. Since the components of the feed have different properties, the ordering forces

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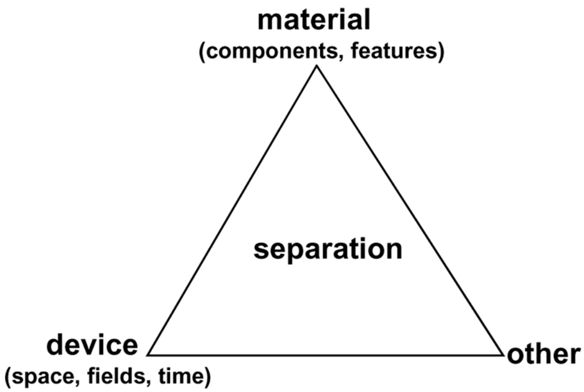


Figure 1. Separation as an interactive system.

acting on them are different in magnitude and therefore the movement of components causes their stratification according to the value of the feature responsible for separation.

The stratification of particles depends not only on ordering but also on other forces operating in separation system. They can be neutral, disordering, fluctuating, etc.

After stratification, the separator has to provide another force or forces which split the stratified material into products. They can be called the splitting forces. Features of particle separation systems are shown in a graphical form in Fig. 2.

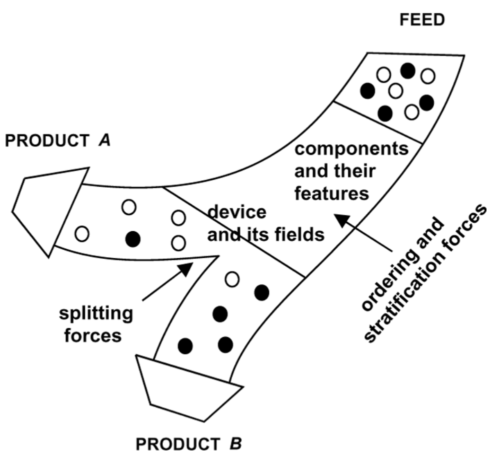


Figure 2. Elements of separation of feed into products.

Since all particle separation systems contain external force fields, which together with the features of the components provide ordering and other forces responsible for stratification as well as contain splitting forces leading to separation products, it should be possible to create a universal way of delineation of separations and prediction of separation results. This is the goal of our work.

It should be added that there are many papers dealing with different aspects of separation. They can be classified, according to Wei and Realff (1), into ideal equilibrium-based separations and non-ideal mechanical separations involving particles. The results of mechanical separations can be modeled and analyzed, according to these authors, either from the probability-based or differential equations-based perspectives. The models of mechanical separations, in most cases, predict either quantity and quality of separation products or quality of separation products related to the numerical values of the feature utilized for separation. Graphical forms of the former approach are known as the Henry, Mayer, Dell, Hall, Fuerstenau, etc. upgrading curves (2) while the latter are classification curves, including the so-called partition (Tromp) curve (3).

The role of this paper is not to compete with the hitherto offered models of separation but rather look at the non-ideal mechanical separation more generally. The proposed universal delineation of separation results is based on the following elements: balance of forces, trajectory of particles, stratification of particles, splitting of stratified particles into products. Depending on the design of separator, the stratification can be performed in a linear, two- and three-dimensional mode. In this work examples of linear (1D) and two-dimensional (2D) stratifications and separations will be considered.

ELEMENTS OF PARTICLE SEPARATION SYSTEMS

Balance of Forces

A particle exposed to external force fields provided by the separator moves in relation to the separator due to appearance of ordering and other forces as a result of particle-field interactions. There are different ordering forces. Some of them are given in Table 1.

Trajectory of a particle in space and time depends not only on the ordering forces but also on other forces operating in the system. Therefore, it is necessary to make a balance of forces which influence the movement of particles in the separator. All forces operating in a separation system are responsible for stratification of particles. Sometimes it is convenient to balance energy instead of force. Other approaches including

Table 1. Examples of ordering forces resulting from interaction of external force fields and components

Main feature of component	Field	Ordering force, F	Remarks
Charge of particle, q	Electromagnetic (electric), E	$F = qE$	
Magnetic susceptibility, χ	Electromagnetic (magnetic), H	$F = \chi\mu_o mH grad\, H$	m = mass, μ_o – vacuum permeability
Hydrophobicity, θ_d (detachment contact angle)	Electromagnetic-derived (surface tension), γ	$F = (1 - cos\,\theta_d)\gamma r$	r = particle radius, $\pi = 3.14$
Mass, m	Gravity, g	$F = mg$	

probability, momentum, etc., are also possible. The choice of property used for balancing and calculating the trajectory of particles depends on individual preferences based on the ease of calculation and other factors. Details of force balancing and other elements of particle separation systems will be discussed in the section titled Examples.

Trajectory of Particle

Appropriate substitution of forces with formulas relating them to physical parameters, coupled with specific, for a given separator requirements (time, space), provide equations of movement of particles in the space as a function of time and particle property.

Stratification

When a system contains more than one element, the elements (particles) undergo stratification due to forces operating in the separator. To determine stratification, the distributions of the main feature of separation system components in the feed have to be known. In many particle separation systems the distribution of the feature utilized for separation is based on probability. In such a case usually normal distribution is used but other distributions can also be used.

Splitting

The stratified material can be subjected to additional force or forces which split the stratified material into products. A formula relating the position of the splitter and composition of the stratified material allows to calculate the results of separation.

Tabulation and Graphical Representation of Results of Separation

It is convenient to present the predicted separation result in a tabular form as well as to plot suitable separation curves. There are many ways of expressing separation results. It includes upgrading (quality vs. quantity), classification (quality vs. feature value), sorting (quality vs. product name), etc. Each approach can be presented by an unlimited number of plots (2).

EXAMPLES

To illustrate principles of universal delineation of separation and its results, 1D and 2D separation cases will be considered.

2D Electrostatic Separation

Urvantsev (4) (Fig. 3) considered electrostatic separation of a feed consisting of hematite and quartz particles which were triboelectrically charged. Each particle was falling along the y axis with a constant speed V_y due to equality of the gravity $F_{g,y}$ and resistance $F_{o,y}$ forces. In the horizontal direction (x) there was an electrical force $F_{el,x}$ acting on the particle due to the electrical field E_{el}. Thus, there was an ordering electrostatic force:

$$F_{el,x} = qE_{el} \tag{1}$$

resulting from a coupling of the electrical field F_{el} and the main particle feature, that is the electrical charge q.

The balance of forces in the y direction was:

$$F_{g,y} = F_{o,y} \tag{2}$$

while in the x direction:

$$F_{el,x}. \tag{3}$$

The physical meaning of the electrical force is:

$$F_{el,x} = qE_{el} = ma. \tag{4}$$

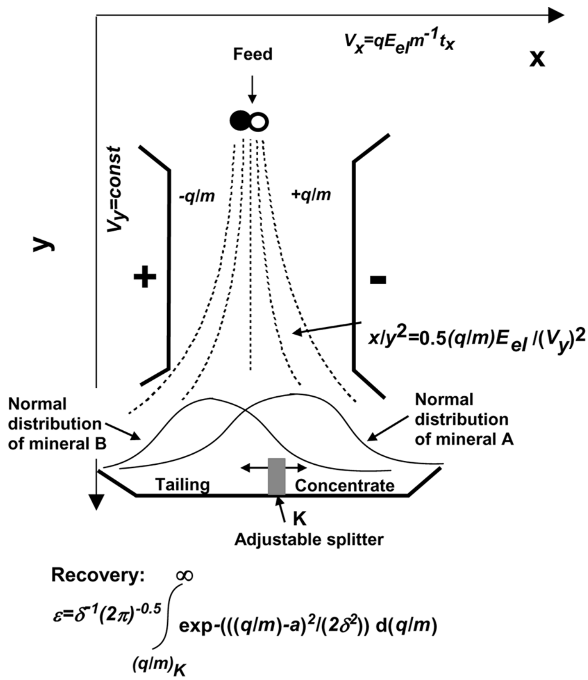


Figure 3. Electrostatic separation of particles. The case considered by Urvantsev (4).

where m stands for particle mass and a for particle acceleration. Therefore, the velocity of the particle in the y direction is given by:

$$V_y = y/t_y = \text{const} \tag{5}$$

while for the x direction is:

$$V_x = at_x = \frac{qE_{el}}{m}t_x \tag{6}$$

where t_x, and t_y denote the time of separation in the indicated direction which is identical with the real time of separation t:

$$t_x = t_y = t. \tag{7}$$

The distance reached by the particle in the x direction will be:

$$x = \frac{1}{2}at_x^2 = \frac{1}{2}\frac{qE_{el}}{m}t_x^2 \tag{8}$$

while for the y direction:

$$y = t_y V_y. \tag{9}$$

A rearrangement and combination of Eqs. (8) and (9) provide the trajectory of a particle indicating that a falling particle will change its position in relation to the x and y axes according to the equations:

$$x = \frac{1}{2}\frac{q}{m}\frac{y^2}{V_y^2}E_{el}. \tag{10}$$

Equation (10) shows that although the main feature utilized for separation is electrical charge of particle q, the separation feature in the considered separator is the electrical charge of particle per particle mass m, that is q/m. The separator facilitates the electrical field E_{el} in the y direction and a constant speed V_{oy} of the falling particle in the x direction as well as the space (x, y) for the falling. The trajectory of falling particle is given by Eq. (10).

In the considered by Urvantsev (4), the separation system of quartz and hematite particles differed in q/m. He assumed that the variation of the q/m followed the normal distribution law. The distribution of mineral A (denoted as h meaning hematite or component 1) which has tendency to be positively charged with the most probable q/m value equal to a_h, is shown in Fig. 3. The probability density function of the random variable q/m is equal to:

$$f_h\left(\frac{q}{m}\right) = \frac{1}{\delta_h\sqrt{2\pi}}\exp\left[-\frac{\left(\frac{q}{m} - a_h\right)^2}{2\delta_h^2}\right], \tag{11}$$

where $f_h(q/m)$ is the probability density function, δ_k^2 – variance of the random variable q/m, a_h – mean value (expected value of q/m for hematite). Similar equations can be written for quartz.

Having the probability density function for both minerals and the position K (Fig. 3) of the splitter it is possible to predict the results of separation.

The equation for recovery of component 1 (hematite) and component 2 (quartz) in a concentrate for a given splitter position K for hematite is given by:

$$\varepsilon_{h,c}\left[\left(\frac{q}{m}\right)_K\right] = \int_{\left(\frac{q}{m}\right)_K}^{\infty} f_h\left(\frac{q}{m}\right) d\left(\frac{q}{m}\right) \tag{12}$$

$$\varepsilon_{h,c} = \frac{1}{\delta_h\sqrt{2\pi}}\int_{\left(\frac{q}{m}\right)_g}^{+\infty} \exp\left[-\frac{\left(\frac{q}{m}-a_h\right)^2}{2\delta_h^2}\right] d\left(\frac{q}{m}\right) = 1-\Phi\left[\frac{\left(\frac{q}{m}\right)_K - a_h}{\delta_h}\right] \quad (13)$$

and for quartz:

$$\varepsilon_{k,c}\left[\left(\frac{q}{m}\right)_K\right] = \int_{\left(\frac{q}{m}\right)_K}^{\infty} f_k\left(\frac{q}{m}\right) d\left(\frac{q}{m}\right) \quad (14)$$

$$\varepsilon_{k,c} = \frac{1}{\delta_k\sqrt{2\pi}}\int_{\left(\frac{q}{m}\right)_K}^{+\infty} \exp -\left[\frac{\left(\frac{q}{m}-a_k\right)^2}{2\delta_k^2}\right] d\left(\frac{q}{m}\right) = 1-\Phi\left[\frac{\left(\frac{q}{m}\right)_K - a_k}{\delta_k}\right] \quad (15)$$

where Φ is the Laplace integral (tabulated).

The position (x, y) of the splitter K fulfils the equation:

$$x = \frac{1}{2}\left(\frac{q}{m}\right)_K \frac{y^2}{V_y^2} E_{el} \quad (16)$$

and provides the value of q/m at that location.

Having recovery for quartz and hematite, a separation curve, for instance the Fuerstenau plot (5), relating the recovery of one component in concentrate versus another component in the same concentrate, can be drawn.

The separation system considered by Urvantsev (4) is summarized in Table 2.

Another example of 2D separation can be the case considered by Brozek and Pawlos (6). They analyzed the separation of particles in a magnetic liquid. Such a separation can be used for diamagnetic materials (nonferrous metals) which differ in densities. Figure 4 shows forces acting on a diamagnetic particle immersed in magnetic liquid and trajectories of particles with density ρ or specific weight γ. Three essential forces, and their components in the x and y directions, are acting on each particle in the separator, that is gravity F_g, magnetic F_m, and resistance F_o forces (Fig. 4). During movement the particle changes its position in relation to both x and y axes due to excess (effective) forces. The balance of forces in the x direction is:

$$F_{ef,x} = F_{g,x}, -F_{o,x} \quad (17)$$

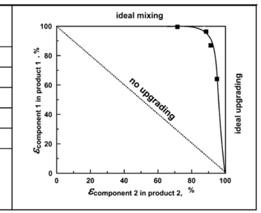
while in the y direction:

$$F_{ef,y} = F_{g,y} - F_m - F_{o,y} \quad (18)$$

Table 2. Elements of delineation of a 2D separation system. The case of electrostatic separation considered by Urvantsev (2) (F_g – gravity, F_o – resistance, F_{el} – electrical forces, E_{el} – electrical field, x – distance, y – distance, V – velocity, m – mass, q – electrical charge, t – time)

Element	Equations
Force balance and particle movement	$F_{g,y} = F_{o,y};\ F_{el,x}\ (V_y = y/t_y = \text{cont};\ V_x = at_x = qE_{el}m^{-1}t_x)$
Condition of separation	$t_x = t_y = t$
Ordering equation (trajectory of a particle)	$x = \frac{1}{2}\frac{q}{m}\frac{y^2}{V_y^2}E_{el}$
Features a) material (main) feature	a) electrical charge, q
b) separation feature in the considered devices	b) electrical charge per unit mass, q/m
External force fields	gravity, electromagnetic
Space and time	$x,\ y,\ t$
Population of particles having certain value of the main feature	$f_i\left(\frac{q}{m}\right) = \frac{1}{\delta_i\sqrt{2\pi}}\exp - \frac{\left(\frac{q}{m} - a_i\right)^2}{2\delta_i^2}$
Splitter position	K
Recovery of a component (here hematite, h)	$\varepsilon_{h,c} = \frac{1}{\delta_h\sqrt{2\pi}}\int_{\left(\frac{q}{m}\right)_K}^{+\infty}\exp\left[-\frac{\left(\frac{q}{m}-a_h\right)^2}{2\delta_h^2}\right]d\left(\frac{q}{m}\right) = 1 - \Phi\left[\frac{\left(\frac{q}{m}\right)_K - a_h}{\delta_h}\right]$
Recovery of remaining components (here quartz, k)	$\varepsilon_{k,c} = \frac{1}{\delta_k\sqrt{2\pi}}\int_{\left(\frac{q}{m}\right)_K}^{+\infty}\exp - \left[\frac{\left(\frac{q}{m}-a_k\right)^2}{2\delta_k^2}\right]d\left(\frac{q}{m}\right) = 1 - \Phi\left[\frac{\left(\frac{q}{m}\right)_K - a_k}{\delta_k}\right]$
Separation balance and separation plot for components 1 and 2 for a given splitter (K) position	(see table and plot below)

Product	yield, γ(%)	content 1, β, %	upgrading ratio β/α	recovery $\varepsilon = \gamma\beta/\alpha$, %
Concentrate 1	12.06	81.70	5.305	63.98
Concentrate 2	20.14	60.40	3.922	79.01
Concentrate 3	42.27	32.44	2.106	89.07
Concentrate 4	70.14	21.73	1.411	98.93
Tailing	29.86	0.52	0.0338	1.01
Feed	100.00	15.40=α	1	100.00



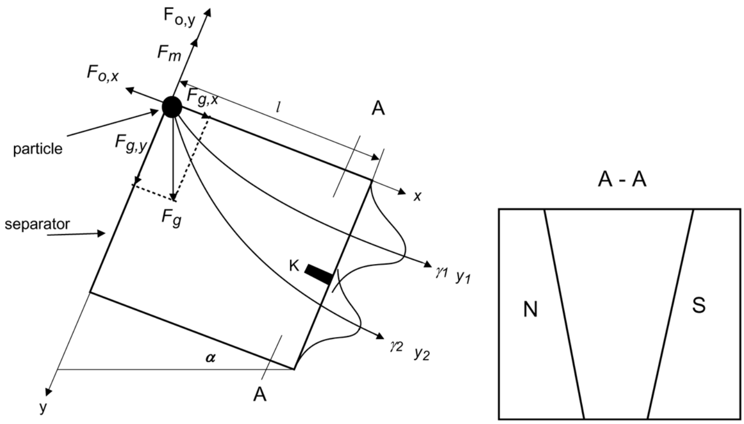


Figure 4. Principles of separation in a magnetic liquid. F_g – gravity force, F_m – magnetic force, F_o – resistance force α – separator inclination (deg), x and y are directions in the space.

The expressions for the forces are:

$$F_{g,x} = (\rho - \rho_o) g V \sin\alpha = (\gamma - \gamma_{og}) V \sin\alpha \tag{19}$$

$$F_{g,y} = (\rho - \rho_o) g V \cos\alpha = (\gamma - \gamma_{og}) V \cos\alpha \tag{20}$$

$$F_{o,x} = k_x b_o V v_x = k_x b_o V dx/dt \tag{21}$$

$$F_{o,y} = k_y b_o V v_y = k_y b_o V dy/dt \tag{22}$$

$$F_m = \mu_o \chi_o V H grad\, H = \gamma_{om} V \tag{23}$$

where ρ stands for particle density, ρ_o magnetic liquid density, γ specific weight of particle ($\gamma = \rho g$), γ_{og} component of liquid specific weight due to gravity ($\gamma_{og} = \rho_o\, g$), χ_o liquid magnetic susceptibility, μ_o magnetic permittivity of vacuum (a universal constant), V particle volume, v particle velocity, α separator inclination, H magnetic field intensity, $gradH$ magnetic field gradient, g acceleration due to gravity, k_x particle shape coefficients for movement along x, k_y particle shape coefficients for movement along y axes, $b_o = 18\eta/d^2$, d particle diameter, η magnetic liquid viscosity, t time, and $\gamma_{om} = \mu_o \chi_o H grad\, H$ is the liquid specific weight component due to magnetic field.

The magnetic separator is designed in such a way that the product of intensity of the magnetic field H and its gradient $grad\, H$ (that is $H grad\, H$)

is a linear function of the position (depths) of any spot in the separator in the y direction. Therefore, the magnetic force is expressed as:

$$F_m = \mu_o \chi_o V(K_a y + K_o) \tag{24}$$

where K_a and K_o are constants. Thus, the expressions for the effective forces acting on a particle in x and y directions are:

$$F_{ef,x} = F_{g,x}, -F_{o,x} = (\gamma - \gamma_{og}) V \sin\alpha - k_x b_o V dx/dt \tag{25}$$

and

$$\begin{aligned} F_{ef,y} = F_{g,y}, -F_m - F_{o,y} &= (\gamma - \gamma_{og}) V \cos\alpha - \mu_o \chi_o V(K_a y + K_o) - k_y b_o V dy/dt \\ &= \gamma V \cos\alpha - \gamma_{oef}(y) V - k_y b_o V dy/dt \end{aligned} \tag{26}$$

where $\gamma_{oef}(y) = \gamma_{og} \cos\alpha + \mu_o \chi_o (K_a y + K_o)$ is the effective specific weight of the liquid in the y direction.

Since, according to Newton's equation, the force is given by:

$$F_{ef,x} = ma_x = \gamma g^{-1} V d^2 x/dt^2 \tag{27}$$

and

$$F_{ef,y} = ma_y = \gamma g^{-1} V d^2 y/dt^2 \tag{28}$$

where m is the mass of particle, the equations for particle movements are:

$$\frac{\gamma}{g}\frac{d^2 x}{dt^2} = \left(\gamma - \gamma_{og}\right) \sin\alpha - k_x b_o \frac{dx}{dt} \tag{29}$$

$$\frac{\gamma}{g}\frac{d^2 y}{dt^2} = (\gamma - \gamma_{og}) \cos\alpha - \mu_o \chi_o (K_a y + K_o) - k_y b_o \frac{dy}{dt} \tag{30a}$$

or

$$\frac{\gamma}{g}\frac{d^2 y}{dt^2} = \gamma \cos\alpha - \gamma_{oef}(y) - k_y b_o \frac{dy}{dt}. \tag{30b}$$

Finally, the equations for particle trajectory, after solving Eqs. (29) and (30) (7) are:

$$x(t) = \frac{(\gamma - \gamma_{og}) \sin\alpha}{b_o^2} \left[k_x b_o t - \frac{\gamma}{g}\left(1 - e^{-\frac{k_x b_o g}{\gamma} t}\right) \right] \tag{31}$$

$$y(t) = \frac{C_1\gamma}{\mu_0\chi_o g K_a} e^{-\frac{k_y b_o g}{2\gamma}t} \sin\sqrt{\frac{4\mu_o K_a \gamma g \chi_o - k_y^2 b_o^2 g^2}{4\gamma^2}}\, t \tag{32}$$

where C_1 is a constant which can be determined taking into account the border conditions. For $v_y(0) = v_{oy}$, therefore:

$$C_1 = \frac{v_o g \cos\alpha}{4\mu_o K_a \gamma \chi_o - b_o^2 k_y^2 g}. \tag{33}$$

Thus, the formula for the particle trajectory in the working space of the separator filled with magnetic liquid having given density ρ_o and magnetic susceptibility χ_o depends on particle density ρ as well as its size and shape and other parameters.

Equations (31–32) are general and can be used for delineation of separation in different practical systems involving magnetic liquids. For instance, for a two-component system in which the particle specific weight γ (in N/m^3) (or density ρ, in kg/m^3, because $\gamma = \rho g$), changes with a normal distribution around average value of $\bar{\gamma}_1$ for component 1 and around $\bar{\gamma}_2$ for component 2 with α, K_a, b_o, k_x, k_y, and χ_o variables being constant. For such a system, similarly to the system considered by Urvantsev, the probability density function f_i of the random variable γ (particle specific weight) is equal to:

$$f_i(\gamma) = \frac{1}{\sqrt{2\pi}\cdot\delta_i} \exp\left[-\frac{(\gamma - \bar{\gamma}_i)^2}{2\delta_i^2}\right] \tag{34}$$

where: δ_i– standard deviation of particle specific weight, $\bar{\gamma}_i$– average value (for normal distribution the most probable) of the considered particle specific gravity ρ measured in specific weight units γ. The symbol i stands for component name (here 1 and 2), and π is 3.14.

Since the densities of component 1 and 2 are different, their recoveries will also be different and, similarly to Eqs. (13) and (15), given by:

$$\varepsilon_1 = \frac{1}{\sqrt{2\pi}\;\delta_1} \int_{(\gamma)K}^{\infty} \exp\left[-\frac{(\gamma - \bar{\gamma}_1)^2}{2\delta_1^2}\right] d\gamma = 1 - \Phi\left(\frac{(\gamma)_K - \bar{\gamma}_1}{\delta_1}\right) \tag{35}$$

and

$$\varepsilon_2 = \frac{1}{\sqrt{2\pi}\;\delta_2} \int_{(\gamma)K}^{\infty} \exp\left[-\frac{(\gamma - \bar{\gamma}_2)^2}{2\delta_2^2}\right] d\gamma = 1 - \Phi\left(\frac{(\gamma)_K - \bar{\gamma}_2}{\delta_2}\right) \tag{36}$$

where Φ denotes the Laplace integral (7).

The summary of the considered 2D magnetic separation is given in Table 3.

In should be noticed that Eqs. (35) and (36) are also valid for the case when the densities of two different components are constant ($\rho_1 = const \neq \rho_2 = const$) but due to fluctuation of the size and shape of the irregular particles and mutual interactions of particles during their movement, the trajectory of each component as well as the particle exit level (measured in specific weight units) is a random variable with a normal distribution. This happens when the source of fluctuations is difficult to express with forces which would be added to the existing forces. In this case the equation is:

$$f_{yi}(y) = \frac{1}{\sqrt{2\pi} \cdot \delta_{yi}} \exp\left[-\frac{(y - \bar{y}_i)^2}{2\delta_{yi}^2}\right] \tag{37}$$

where: f_{yi} is the probability density function, δ_{yi} – standard deviation of variable y (exit level), $\bar{y}_i$ – average value (for normal distribution the most probable) of the level of particle exit for a considered particle specific gravity $\rho = \text{const}$ measured in specific weight units γ ($\gamma = \rho g$). The symbol i stands for component name (here 1 and 2) while π is 3.14. This approach, however, represents another, purely probabilistic, treatment of the separation results and will be considered in other publications.

Literature on modeling of 2D separation is extensive and includes, for instance, already mentioned accomplishments of Wei and Realff (1,8).

1D Separation

Drzymala (9) (Fig. 5) considered a simplified (1D) case of particles separation by flotation. During flotation such parameters as particle hydrophobicity θ, particle density ρ_p, density of liquid ρ_w, bubble size R, surface tension γ_{lv} were constant. In a given experiment the size of all particles was the same but in different experiments the size of particles was varied. The particles were split into floating and sinking products. At certain radius of particles ($r_{p\ max}$) the yield of floating particles is exactly 50% because at this weight of the particle, according to the probability theory, 50% of them are too heavy, detaches from the bubble surface, and sinks. This is the base of the so-called flotometry that can be used for determination of hydrophobicity (contact angle θ) of particles.

The balance of force is (Fig. 6):

$$F_{up} = F_{down} \tag{38}$$

Table 3. Elements of delineation of separation in a magnetic liquid as an example of 2D separation system considered by Brozek and Pawlos (6). Meanings of symbol are explained in the text

Element	Equations
Force balance and particle movement	$F_{ef,x} = F_{g,x'} - F_{o,x}$ $F_{ef,y} = F_{g,y'} - F_m - F_{o,y}$ and $F_{ef,x} = ma_x = \rho V d^2 x / dt^2$ $F_{ef,y} = ma_y = \rho V d^2 y / dt^2$
Ordering equation (trajectory of a particle)	$x(t) = \frac{(\rho - \rho_0) g \sin\alpha}{b_o^2}\left[k_x b_o t - \rho\left(1 - e^{-\frac{k_x b_o}{\rho} t}\right)\right]$ $y(t) = \frac{C_1 \rho}{\mu_0 \chi_o K} e^{-\frac{k_y b_o}{2\rho} t} \sin\sqrt{\frac{4\mu_o K_a \rho \chi_o - b_o^2}{4\rho^2}}\, t$
Features a) material (main) feature) b) separation feature in the considered devices	density ρ (or equivalent, here specific weight $\gamma = \rho g$) density ρ (or equivalent, here specific weight $\gamma = \rho g$)
External force fields	gravity, electromagnetic (magnetic and resistance)
Space and time	$x,\ y,\ t$
Distribution of particles having certain value of the main feature	$f_i(\gamma) = \frac{1}{\sqrt{2\pi} \cdot \delta_i} \exp\left[-\frac{(\gamma - \bar{\gamma}_i)^2}{2\delta_i^2}\right]$
Splitter position	$K\ (x,y)$ at $x = l$
Recovery of one component	$\varepsilon_1 = \frac{1}{\sqrt{2\pi}\,\delta_1} \int_{(\gamma)_K}^{\infty} \exp\left[-\frac{(\gamma - \bar{\gamma}_1)^2}{2\delta_1^2}\right] d\gamma = 1 - \Phi\left(\frac{(\gamma)_K - \bar{\gamma}_1}{\delta_1}\right)$
Recovery of remaining components	$\varepsilon_2 = \frac{1}{\sqrt{2\pi}\,\delta_2} \int_{(\gamma)_K}^{\infty} \exp\left[-\frac{(\gamma - \bar{\gamma}_2)^2}{2\delta_2^2}\right] d\gamma = 1 - \Phi\left(\frac{(\gamma)_K - \bar{\gamma}_2}{\delta_2}\right)$
Separation balance and plot	see Table 2

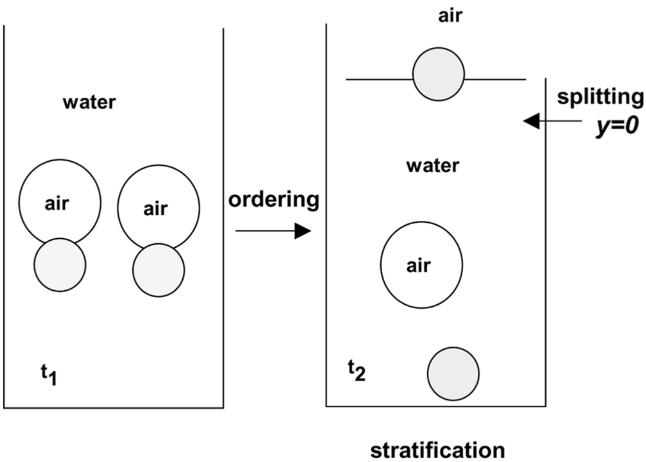


Figure 5. Flotational separation considered in this work. At certain size of the hydrophobic particle (having hydrophobicity θ and other parameter constant) there is an equal chance for the particle either to sink or float. Then, the yields of sink and float products are 50%.

$$F_{up} = F_{c\,(\max)} + F_v + F_h \tag{39}$$

$$F_{down} = F_g + F_p \tag{40}$$

thus:

$$F_{c(\max)} + F_v + F_h = F_g + F_p \tag{41}$$

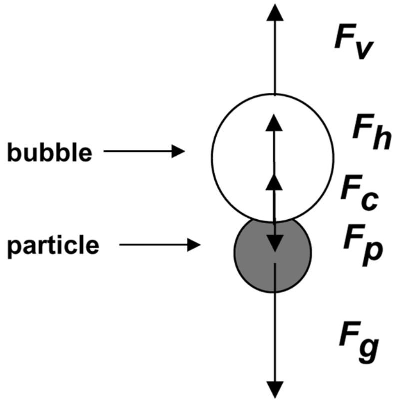


Figure 6. Balance of forces in flotometry.

where:

$F_{c(max)}$ – maximum capillary force at the moment of particles detachment from the bubble
F_g – weight force of particle in the air
F_v – buoyancy force (weight of particle in water is: $F_g - F_v$)
F_h – hydrostatic pressure force acting on grain at the point of bubble attachment
F_p – additional pressure inside bubble.

The forces are given by the equations:

$$F_{c(\max)} = \pi r_p \gamma_{lv}(1 - \cos\theta_d), \tag{42}$$

$$F_g = (4/3)\pi r_p^3 \rho_p g, \tag{43}$$

$$F_v = \pi r_p^3 \rho_w g\{(2/3) + \cos(\theta_d/2) - (1/3)\cos^3(\theta_d/2)\}, \tag{44}$$

$$F_p - F_h = \pi r_p^2(1 - \cos\theta_d)(\gamma_{lv}/R - R\rho_w \mathrm{g}). \tag{45}$$

Therefore, the final relation is:

$$\begin{aligned}&\pi r_p \gamma_{lv}(1 - \cos\theta_d) - [(4/3)\pi r_p^3 \rho_p g - \pi r_p^3 \rho_w g\{(2/3) + \cos(\theta_d/2)\\ &- (1/3)\cos^3(\theta_d/2)\}] - \pi r_p^2(1 - \cos\theta_d)(\gamma_{lv}/R - R\rho_w g) = 0,\end{aligned} \tag{46}$$

in which, due to geometry of the system:

$$\theta = \text{arc}\ \sin(r_{p\,max} R^{-1} \sin(\theta_d/2)) + \theta_d/2 \tag{47}$$

In the equations r_p – particle radius, γ_{lv} – liquid surface tension, ρ_p – particle density, ρ_w – water density, g – acceleration due to gravity, θ_d – the angle of detachment of particles from bubble, θ – contact angle (a measure of hydrophobicity), R – bubble radius, $\pi = 3.14$.

Equation (46) determines the trajectory of particles. It contains the main parameter r_p, constant parameters (θ, ρ_p, R), and external force fields provided by the device (g, γ_{lv}). For the left hand term greater than zero, the particle will float and its location, on the y axis, is positive while for the left hand term smaller than zero the particle sinks (location of particle y is negative).

Similar 1D cases can be found in literature. For instance Nesset and Finch (10) considered separation of particles in a Franz isodynamic separator. Having the magnetic field at which the yield of particles was 50%, they calculated the magnetic susceptibility (main feature) of hematite.

Table 4. Elements of delineation of a 1D separation system. The case of flotational separation considered by Drzymala (9)

Element	Equations
Force balance	$F_{up} = F_{down}$
Condition of separation	$r_p = r_{p\ max}$ providing yield of floating particles = 50%, $y = 0$
Ordering equation (trajectory of a particle)	trajectory (up: $y = +$ or down: $y = -$). $y = 0$ for $r_p = r_{p\,max}$ given by the equation: $\pi r_p \gamma_{lv}(1 - \cos\theta_d) - [(4/3)\pi r_p^3 \rho_p g - \pi r_p^3 \rho_w g\{(2/3) + \cos(\theta_d/2) - (1/3)\cos^3(\theta_d/2)\}] - \pi r_p^2 (1 - \cos\theta_d)(\gamma_{lv}/R - R\rho_w g) = 0$, $\theta = \arcsin(r_{p\,max} R^{-1} \sin(\theta_d/2)) + \theta_d/2$, providing yield of floating particles = 50%
Features	in this case particles size (r_p)
a) material (main) feature	particles size (r_p) (other parameters such as θ, ρ, R, γ, etc. are constant)
b) separation feature in the considered case and device	
External force fields	gravity, electromagnetic (buoyancy, hydrostatic, capillary, additional bubble pressure)
Space and time	y (1D), t not a parameter
Distribution of particles having certain value of the main feature	identical particles (no distribution) in each experiment
Splitter position	fixed near the water-gas interface of the flotation cell
Recovery of one component	$\varepsilon = 50\%$ (component: floating particles) for a suitable particle size for given particle hydrophobicity (θ) and density as well as water density, bubble size etc.
Recovery of remaining components	$\varepsilon = 50\%$ (component: non-floating particles)
Separation balance and plot	not needed or recovery of floating particles as a function of particles size to determine the size at which the recovery is 50%

CONCLUSIONS

Particle separation systems and processes, regardless of complexity consists of universal elements. The most important are the balance of forces (or equivalent quantities such as energies, momentum, probabilities, etc.), particle trajectory, stratification of particles, and splitting the stratified material into products. Defining these element makes the delineation of separation and prediction of separation results of stratified material easy to handle.

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